Quantitative Literacy:
Thinking Between the Lines

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Chapter 9: Geometry

Chapter 9 Geometry
Lesson Plan

- Perimeter, area, and volume: How do I measure?
- Proportionality and similarity: Changing the scale
- Symmetries and tilings: Form and patterns

Learning Objectives:

- Various types of symmetries: in art, architecture, and nature
  - *Rotational symmetries* in the plane
  - *Reflectional symmetry* of planar figures
  - Regular tilings of the plane
  - Irregular tilings

9.3 Symmetries and tilings: Form and patterns

- A planar figure has **rotational symmetry** about a point if it remains exactly the same after a rotation about that point of less than 360 degrees.

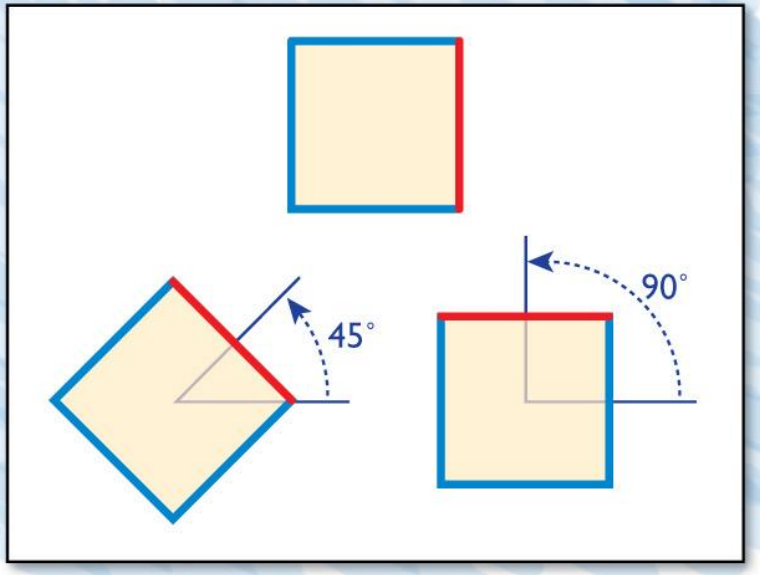


FIGURE 9.47 A square has 90-degree rotational symmetry but not 45-degree rotational symmetry.

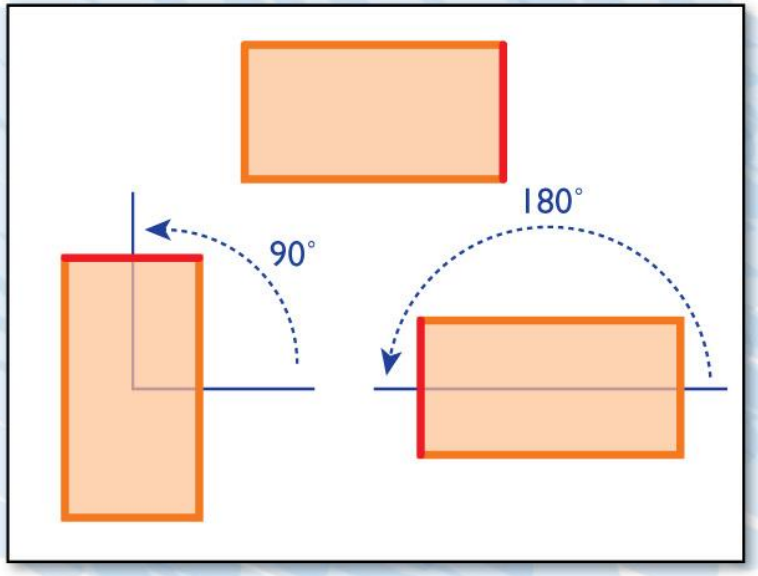


FIGURE 9.48 A rectangle has 180-degree rotational symmetry but not 90-degree rotational symmetry.

- **Example:** Find the rotational symmetries of the five-pointed star, or pentagram, in Figure 9.50.

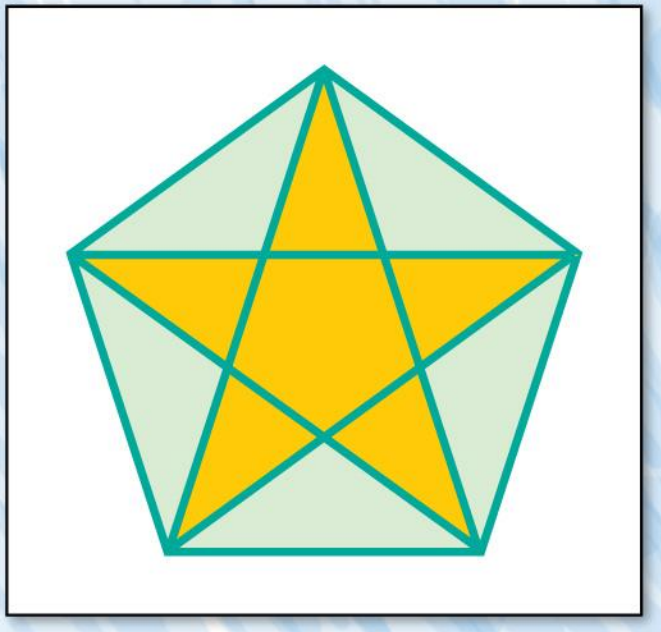


FIGURE 9.50 A pentagram.

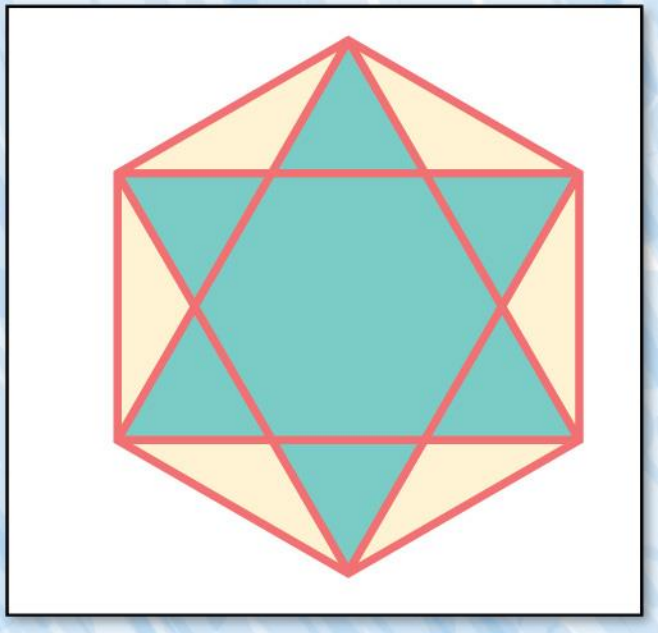


FIGURE 9.51 A six-pointed star.

- **Solution:** In order to preserve the star, we must rotate through an angle that takes one star point to anther.

  The star points divide 360 degrees into five equal angles:

$$\text{A rotation of } \frac{360 \text{ degrees}}{5} = 72 \text{ degrees moves one star point to the next}$$

  The pentagram has 72-degree rotational symmetry about its center.

  The other rotational symmetries are multiples of 72 degrees:

$$2 \times 72 \text{ degrees} = 144 \text{ degrees}$$

$$3 \times 72 \text{ degrees} = 216 \text{ degrees}$$

$$4 \times 72 \text{ degrees} = 288 \text{ degrees}$$

9.3 Symmetries and tilings: Form and patterns

- A planar figure has **reflectional symmetry about a line** *L* if the figure is identical to its reflection through *L*.

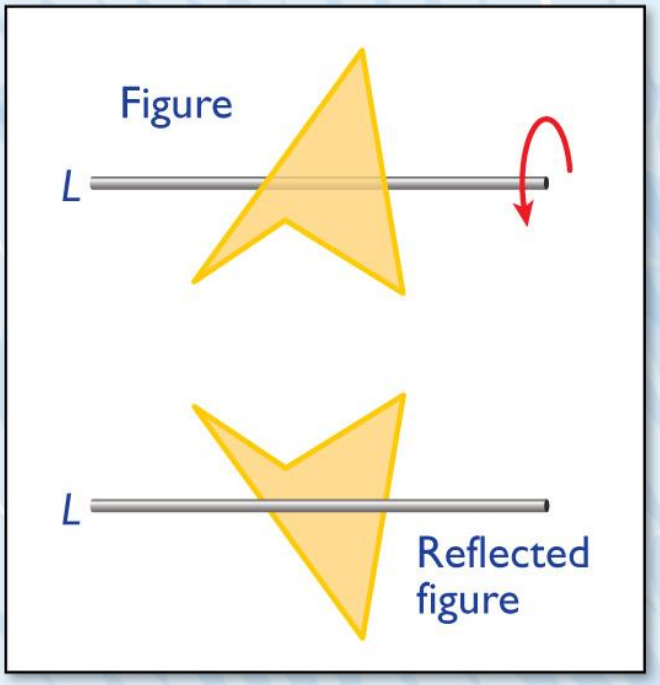


FIGURE 9.52 Reflection is the same as flipping the plane about a line.

9.3 Symmetries and tilings: Form and patterns

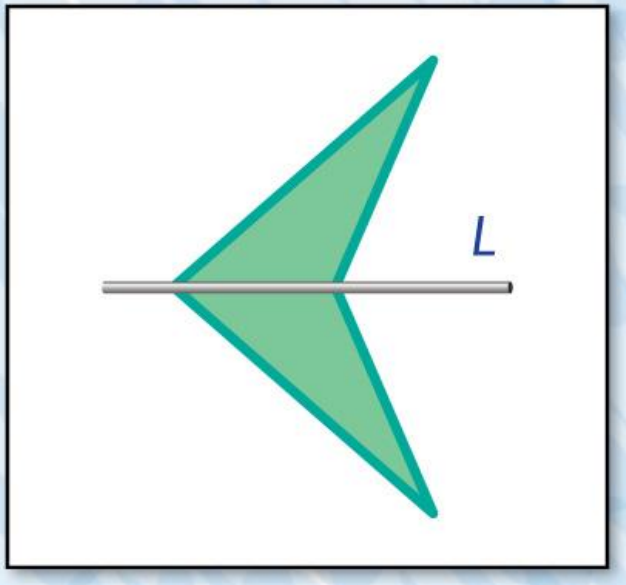


FIGURE 9.53 This shape has reflectional symmetry about the line shown.

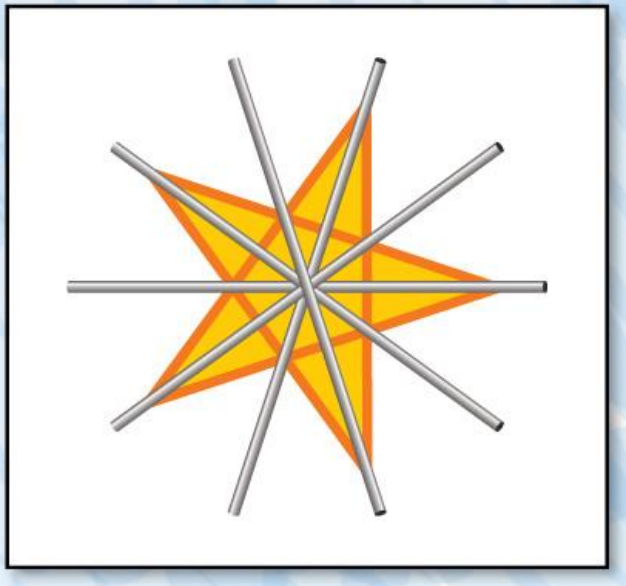


FIGURE 9.54 This star has reflectional symmetry about five different lines.

9.3 Symmetries and tilings: Form and patterns

- **Example:** What are the rotational and reflectional symmetries of the shape in Figure 9.55?

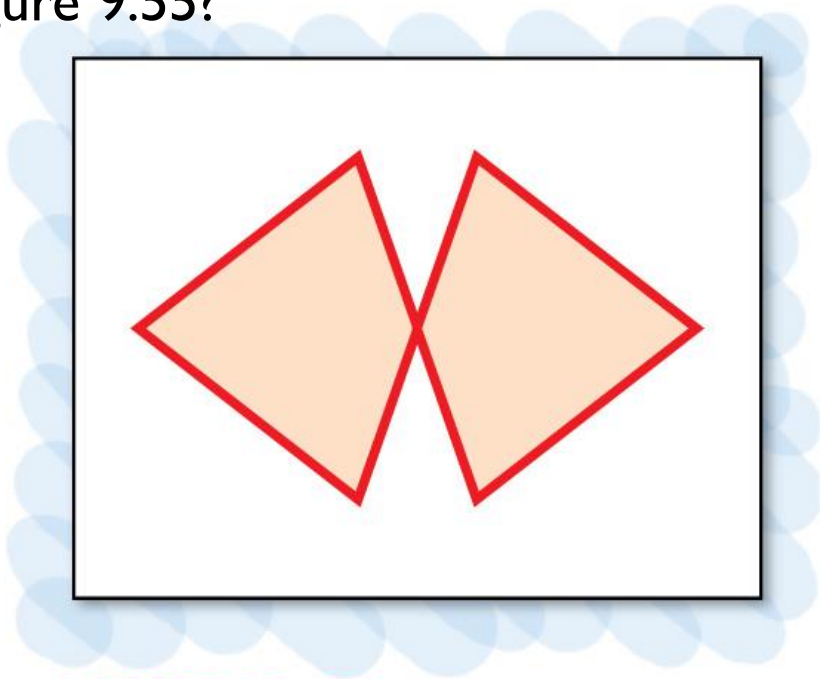


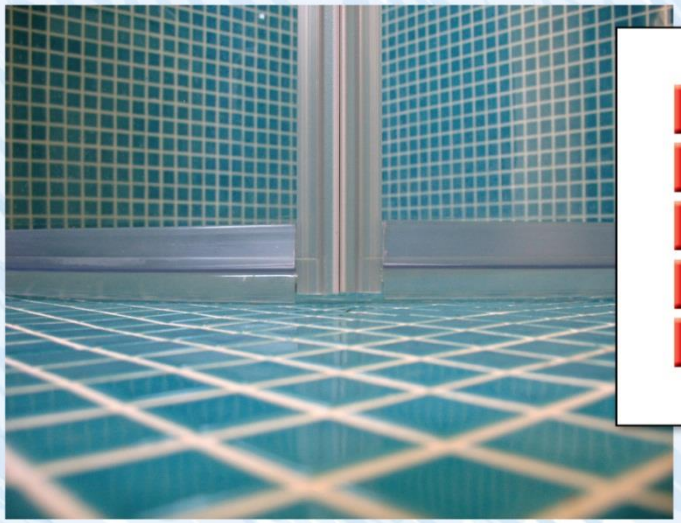
FIGURE 9.55

- **Solution:** There is reflectional symmetry about both horizontal and vertical lines.

  The shape has a rotational symmetry of 180 degrees.

9.3 Symmetries and tilings: Form and patterns

- A **tiling** or **tessellation** of the plane is a pattern of repeated figures that cover up the plane.



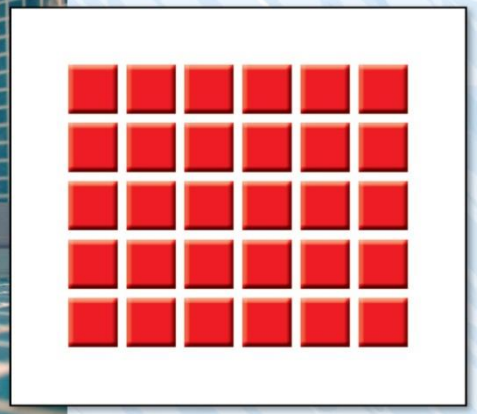


FIGURE 9.57 A tiling by squares.

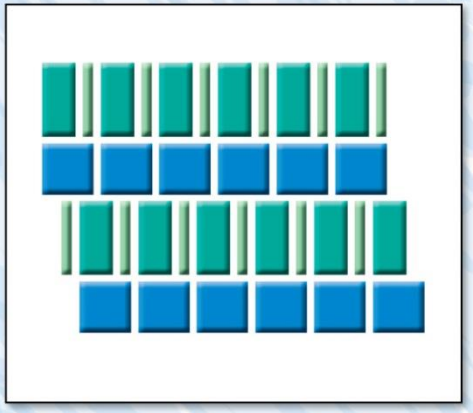


FIGURE 9.58 A tiling by squares and two rectangles.

9.3 Symmetries and tilings: Form and patterns

A **regular tiling** is a tiling of the plane that consists of repeated copies of a single regular polygon, meeting edge to edge, so that every vertex has the same configuration.

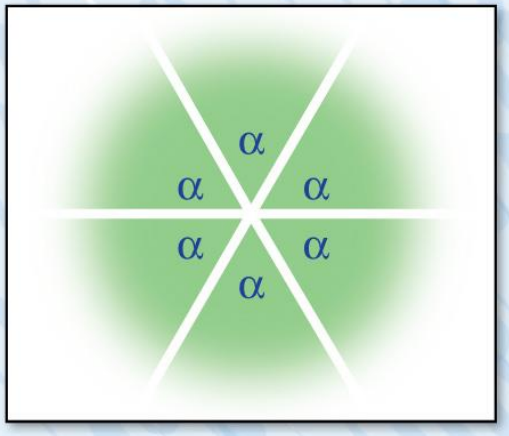


FIGURE 9.59 In a regular tiling, the angles that meet at a vertex must sum to 360 degrees. If there are six polygons, the angles must be 60 degrees.

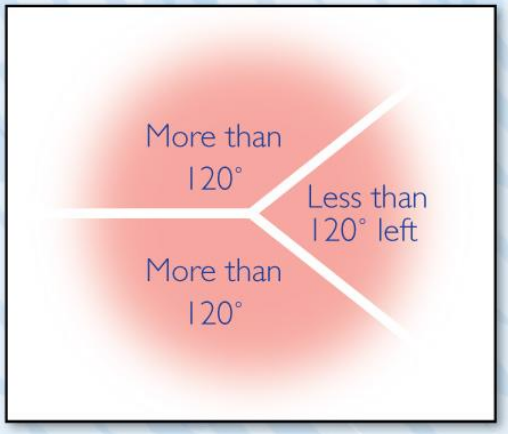


FIGURE 9.60 If a polygon has an angle of more than 120 degrees, a regular tiling cannot be created.

9.3 Symmetries and tilings: Form and patterns

A *regular polygon* is a polygon (a closed figure made of three or more line segments) in which all sides are of equal length and all angles have equal measure.

Regular polygon	Angle size
Equilateral Triangle	60^0
Square	90^0
Pentagon	108^0
Hexagon	120^0

- **Example:** Show how to use an equilateral triangle to make a regular tiling of the plane.
- **Solution:** The tiling is shown in Figure 9.61.

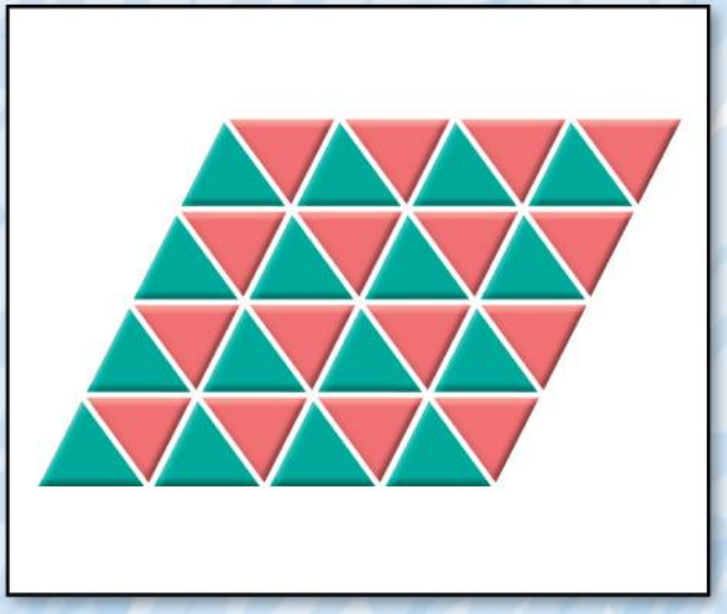


FIGURE 9.61 A regular tiling with equilateral triangles.

9.3 Symmetries and tilings: Form and patterns

Irregular tilings are common in art and architecture.

- **Regular tilings** must satisfy two conditions:

1. They use a single regular polygon, and the same configuration of edges must occur at each vertex.
2. Tilings that are not regular fail to meet one of these two conditions.

One classic example of a tiling that is not regular is shown in the accompanying photo of floor tiles. This tiling does satisfy the second condition, so it is *semi-regular*.

9.3 Symmetries and tilings: Form and patterns

- Figure 9.62 shows how to cut a regular hexagon into three (non-regular) pentagons. The plane can be tiled by regular hexagons.

  Then such a tiling automatically gives an irregular tiling by pentagons.

FIGURE 9.62 Cutting a regular hexagon into three (irregular) pentagons gives an irregular tiling.

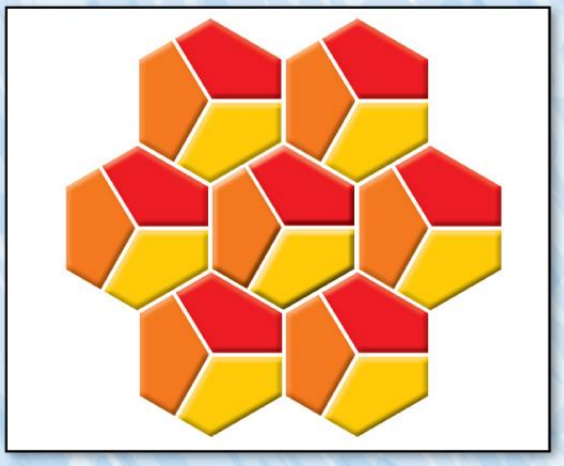


FIGURE 9.63 Using a hexagon tiling to make a pentagon tiling.

9.3 Symmetries and tilings: Form and patterns

- The idea, suggested by Figure 9.63, that we can make new tilings by subdividing old ones is fruitful. We will use it to show how to use any triangle to tile the plane. We begin with the triangle shown in Figure 9.64.
  1. We put together two copies of this triangle to make a parallelogram, as shown in Figure 9.65.
  2. Now we use parallelograms to tile the plane as shown in Figure 9.66 and this gives the required tiling by triangles.

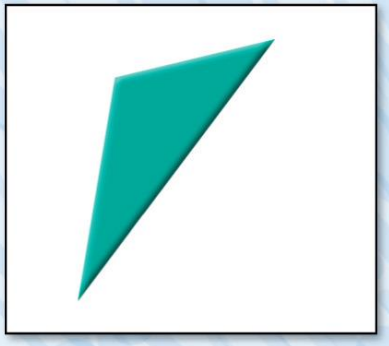


FIGURE 9.64 A triangle.

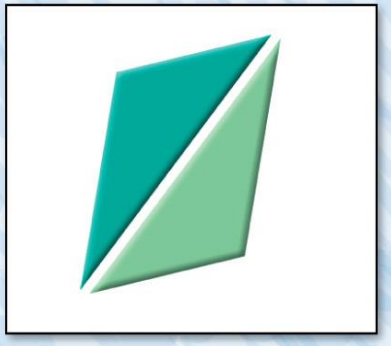


FIGURE 9.65 Two copies of a triangle make a parallelogram.

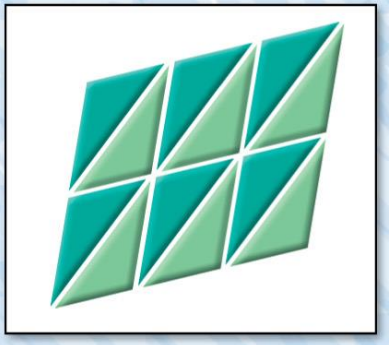


FIGURE 9.66 Using the parallelogram to make a triangle tiling.

- **Example:** Show that the three pieces in Figure 9.67 can be used to tile the plane.

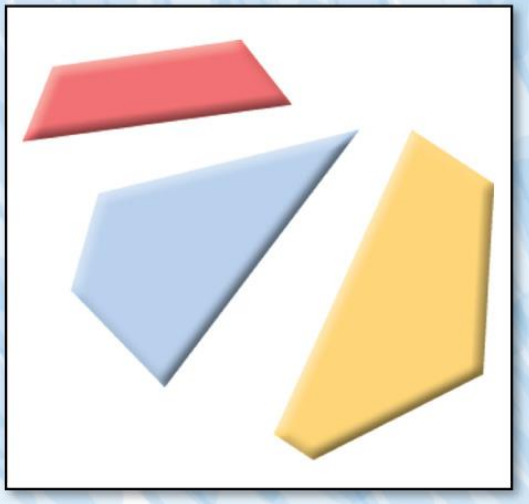


FIGURE 9.67 Pieces for Example 9.18.

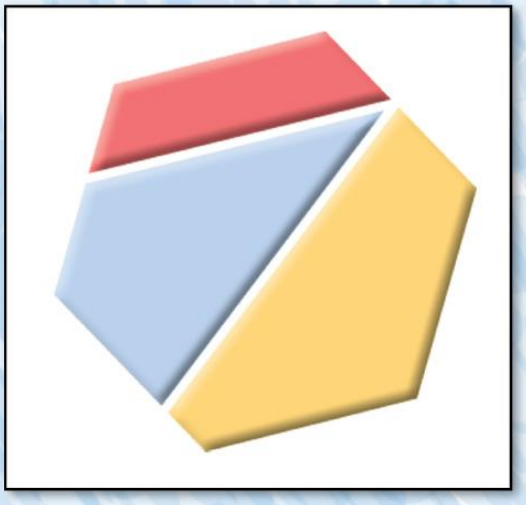
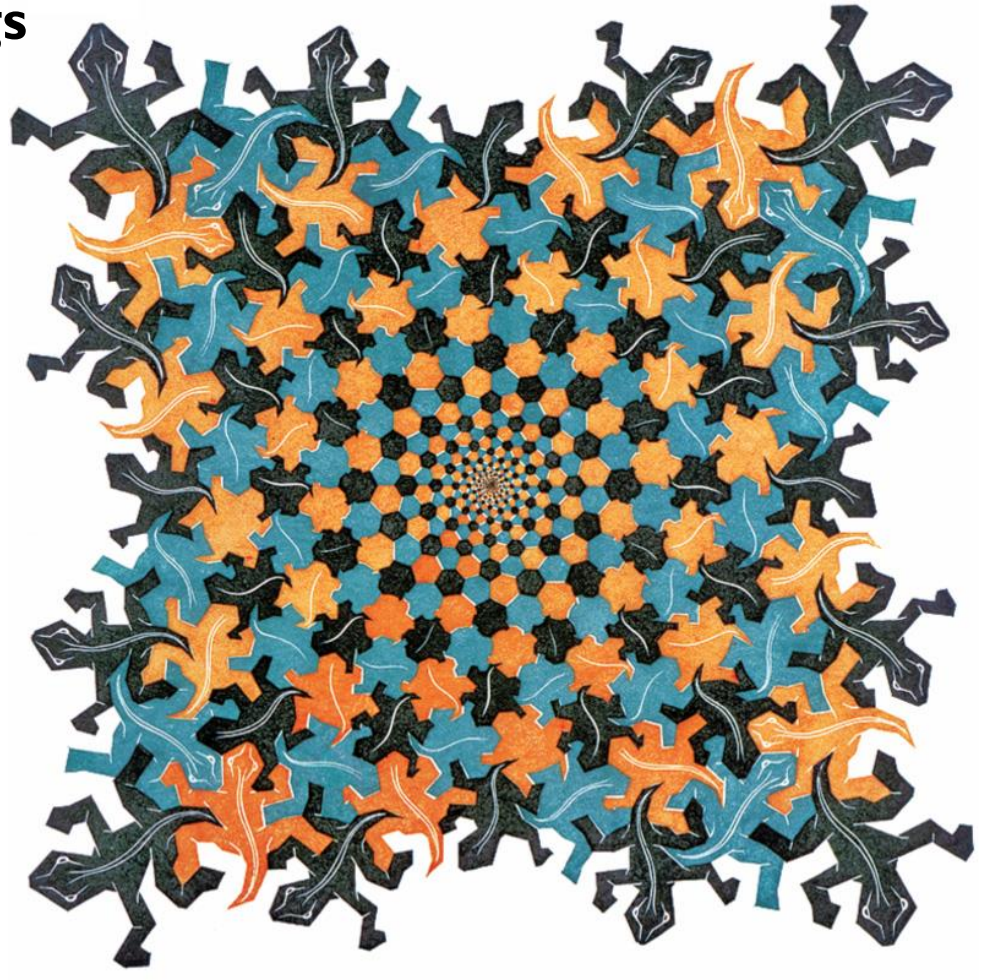


FIGURE 9.68 Pieces go together to make a regular hexagon.

- **Solution:** The three pieces go together to make the regular hexagon shown in Figure 9.68. We know that regular hexagons tile the plane, and that gives a tiling using the three pieces shown.

9.3 Symmetries and tilings: Form and patterns

- **Escher tilings**



Some famous Escher prints.

- **Regular tiling:** From each square, remove a puzzle tab from the bottom and add it to the top of the square.
- The resulting tiling piece is the irregular shape shown at the bottom of Figure 9.70. We get the new tiling of the plane.

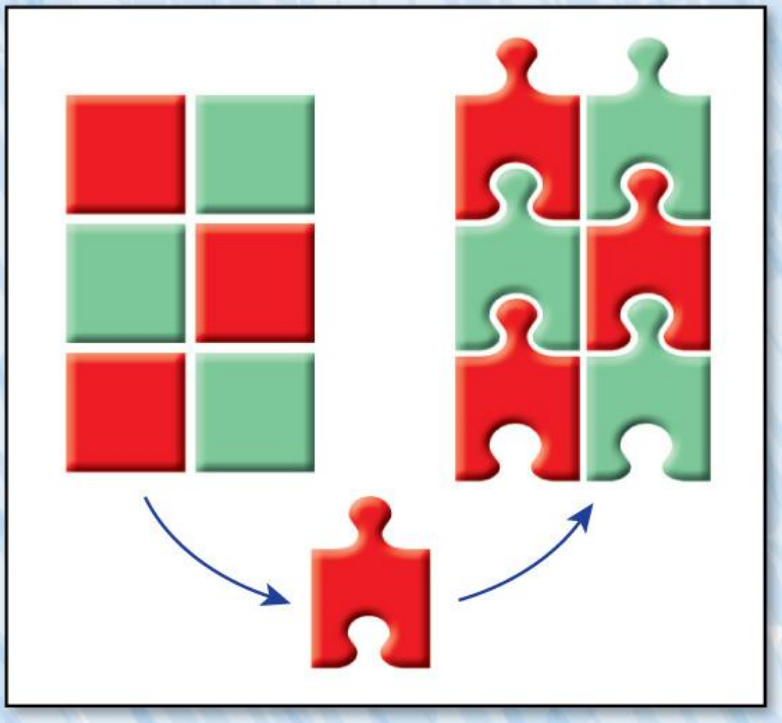


FIGURE 9.70 A regular tiling by squares changed into an irregular tiling.

Chapter 9 Geometry: **Chapter Summary**

- **Perimeter, area, and volume:** How do I measure?
  - Geometric objects can be measured in terms of perimeter (circumference), area and volume.
  - The Pythagorean Theorem of a right triangle
- **Proportionality and similarity:** Changing the scale
  - Proportional and its constant of proportionality
  - Golden rectangles
  - Similar triangles
- **Symmetries and tiling:** Form and patterns
  - Rotational symmetry and reflectional symmetry about a line
  - Tiling or tessellation
  - Regular tiling